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way to the fashioning of an inclusive social ideal—not any more, obviously, than he can as an individual have aught to do with the standards by which the truth of his own factual judgments is to be measured. The problem presented by personal idealism is then just the problem of combining with idealism a conception of personality which has genuinely positive meaning. For in a society with one "perfect" member there can be no personality—not in the perfect member because his perfection consists in a fatal "emancipation" from those so-called "limitations" which give any genuine personality its life and meaning; and not in the others because, measured by the supposed perfections of the preëminent One, they must dwindle to nothingness. Until personal idealism has learned to define human personality in the empirical and human terms of impulse, feeling, sympathy, and purposive rationality, instead of in terms which are the negation of a negation of what personality actually is, it will, I think, contribute little to the clarification of present issues in philosophy. But when it has done this it will have ceased to be idealism in any current meaning of the term.

H. W. STUART.

STANFORD UNIVERSITY.

Principia Mathematica. By ALFRED NORTH WHITEHEAD and BERTRAND RUSSELL. Vol. I. Cambridge University Press, 1910.—pp. xiv, 666.

This is the first volume of a book which students of mathematical logic have—not without some trepidation—been expecting since the publication of Russell's *Principles of Mathematics* in 1903. In the preface to the last named book Mr. Russell promised to give us, with the coöperation of the ingenious author of the *Universal Algebra*, a second volume wherein all the theses of the first volume would be demonstrated "with all the certainty and precision of which mathematical demonstrations are capable." The earlier book, however, left Mr. Russell involved in several hopeless contradictions as to the nature of classes; and as the authors believe that they are now able, by means of the theory of types, to avoid all self-contradiction, they have deemed it preferable to make the present work entirely independent of the *Principles of Mathematics.*

In this, and in the two volumes which are to follow, the authors aim to give us, in strictly mathematical form, a complete systematic deduction of all the fundamental principles of mathematics from a number of primitive propositions which are clearly logical in their character, *i. e.*, propositions which are simply rules of inference. To facilitate this work they have adopted an elaborate system of symbolism, based mainly on the work of Peano, and the body of the book is written exclusively in this symbolic language (excepting two primitive propositions which cannot be symbolically expressed). As a concession to the general or non-mathematical reader, we have, besides an introduction of 88 pages, summaries at the beginning of each part, section, and chapter; and even in the body of the text there is an occasional interpolation in English. This policy of concession, however, involving as it does the occasional 'sacrifice of correctness to lucidity,' seems to be of doubtful wisdom. The repetition of explanations in different forms is bound to cause some confusion. Indeed, the careful reader might do well to omit the introduction altogether. The explanation of the symbolism can perhaps be more clearly gathered from the body of the book, *i. e.*, from the explanation of the symbols as the need for their use arises. The elaborate explanation of the theory of types[1] in the introduction seems needlessly complex. At any rate it is put far more clearly and with equal accuracy in Mr. Russell's article in the *American Journal of Mathematics* of July, 1908.

Part I (pp. 91–342) is devoted to the development of mathematical logic, and the first section of it is naturally devoted to the theory of deduction. The theory of propositions is taken as primary, and by means of it and the theory of apparent variables, the theory of classes, as well as the logic of relations, is deduced. The logic of relations, being of the greatest importance for mathematics, receives the largest share of attention. A good deal is also made of the logic of descriptive phrases such as the 'author of Waverly.' It is pointed out that while that phrase and the word 'Scott' denote the same object they are not equivalent in meaning and hence the statement 'Scott is the author of Waverly' is significant. While the authors do not refer to the general problem of predication in the form in which it disturbed the Megarians and Plato, their theory of predicative functions will be found highly suggestive by those who have dealt with the problem in its traditional form.

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[1] I have refrained from examining here this theory of types because the subject is of considerable importance and I could not, with the limited space at my disposal, deal adequately with it. Besides, there is an admirable account of this logical theory by Dr. H. C. Brown in the *Journal of Philosophy*, etc., Vol. VIII, p. 85 f. It is to be noted, however, that the whole theory is not in the strictest sense necessary for the main thesis of this work. Its office is simply negative. "It forbids certain inferences which would otherwise be valid, but does not permit any which would otherwise be invalid." Hence the authors can say: "Hardly anything in our book would be changed by the adoption of a different doctrine of types." Any theory which accomplishes what this does, viz., enables us to construct a mathematical logic which does not lead to contradiction, would serve the purpose equally well.

Part II (pp. 345–688) is entitled Prolegomena to Cardinal Arithmetic, but is essentially a continuation of Part I. The numbers 1 and 2 are introduced in the first section, not, however, as cardinal numbers, but as unit classes and couples. Other sections are devoted respectively to (*B*) sub-classes, sub-relations and relative types, (*C*) one-many, many-one, and one-one relations, (*D*) selections, and (*E*) inductive relations. The last section deals with certain general ideas of which mathematical induction is a particular instance.

Having thus laid the basis for the theory of finite and infinite series, we are prepared for the definition of cardinal numbers with which Vol. II is to open.

The printing of this volume, involving so much unusual symbolism, is highly creditable to the resources of the Cambridge University Press and to the patience and care of those who read the proofs. There are relatively few misprints and those of a kind which can readily be corrected by the reader. A few slips in the introduction might perhaps cause some confusion. Thus on p. 19, third line from the bottom, $\phi\hat{z}$ should be printed $\phi\hat{x}$. On p, 22, ll. 18 and 20, the implication signs are omitted. On p. 34, l. 20, yRx should be xRy. On p. 32, l. 10, ϕx would be more in consonance with the subsequent interpretation than ϕc.

Apart from the general interest of its main thesis, and the questions aroused by the new logical theory of types, the present volume seems to touch philosophic interests in fewer points than did the *Principles of Mathematics*. The authors have in large measure succeeded in "avoiding both controversy and general philosophy." The intellectual realism which made the reading of the *Principles of Mathematics* so exhilirating is here considerably softened down. Propositions, for instance, are no longer spoken of as entities, but as incomplete symbols having a meaning in use but not in isolation (pp. 46 and 169). Chapter 20 ventures no opinion as to whether a class has in any sense an existence as one object. In the symbolic form of exposition such apparently wild assertions as "false propositions imply all propositions," or "any proposition implies itself," lose their startling character and appear as quite natural and tame. There is also, in this maturer book, less attempt at any *tour de force*. Thus, instead of attempting to define negation and disjunction in terms of implication, as is done in the *Principles of Mathematics*, negation and disjunction are here assumed as primitive, and implication is defined in terms of these two.

On the whole one misses the exuberance and suggestiveness of the earlier book, but the loss seems offset by the feeling that the results of the present work are more mature and reliable.

There are some objections, or rather misapprehensions as to the nature of logistics, which the present book will probably help to remove. In the first place there is the objection of Pesloüan and others that in it simple ideas are defined in terms of very complex ones. Thus Poincaré does not refrain from criticizing Burali-Forti's definition of 1 because it is not calculated to give an idea of that number to one who has not heard of it before (*Revue de Métaphysique et de Morale*, Vol. 13, p. 823). The answer which a careful reading of this volume suggests is that the more familiar is not necessarily the simpler. At any rate there is no question in this volume as to the absolute simplicity of ideas. All that is considered is the question how the whole system of ideas called mathematics can be built up with the smallest number of undefined elements. Our authors succeed in doing so with a very small number, but make no claim that that number might not possibly be reduced.

A second objection has been urged by Kerry, Hilbert, Natorp, and others, to the effect that the simple logical ideas from which the ordinary mathematical concepts are developed, already presuppose those very mathematical concepts. This objection presupposes something like an absolute order of priority or presupposition in ideas, a conception of doubtful validity. Our authors, however, are at pains to indicate that they make no claim to any absolute priority for their primitive ideas, or to any absolute certainty for their primitive propositions. All that they claim—and this they amply prove—is that these ideas and propositions are *sufficient* to build up the whole realm of pure mathematics.

As a result of the rigidly demonstrative form of this volume, the main thesis, viz., that pure mathematics is symbolic logic, *i. e.*, that mathematical propositions are essentially rules of inference, emerges here even more cogently than it did in the *Principles of Mathematics.* This thesis seems to the reviewer of the utmost importance for the theory of logic and metaphysics. If taken seriously, it must lead to a revision of the inadequate dogma that in demonstrative reasoning there is nothing in the conclusion which is not already contained in the premises. Any modification or revision of this dogma will necessarily lead to a new questioning of hearts concerning the easily repeated but essentially obscure doctrine that all truth is derived from experience.

It is to be hoped that the completion of this monumental work will help to dignify a subject which it has been the fashion of philosophers since Lotze to treat with more or less derision. Whatever objections we may have to symbolic logic, we must admit that here it becomes a powerful instrument for the analysis of actual mathematical procedures. It enables us to generalize such notions as *addition* and *multiplication*, and makes such concepts as continuity and infinity subjects of investigation and determination, instead of objects of helpless awe or the recurrent occasions for intellectual violence. On the whole the authors of this work seem to have shown "that with the aid of symbolism deductive reasoning can be extended to regions of thought not usually supposed amenable to mathematical treatment."

Doubtless there will be mathematicians and logicians who will point out diverse errors in the various demonstrations, and philosophers who will find fault with the presuppositions and general methods of this work. To all those, however, who value exactness of thought for its own sake, this volume and the stupendous labor which it expresses, will appeal as a monument of devotion to pure thinking. To those who are inclined to belittle the value of such work it may not be amiss to repeat the words of Novalis; "Das Leben der Götter ist Mathematik. . . . Reine Mathematik ist Religion. Die Mathematiker sind die einzig Glücklichen." This sounds like romanticism but it is essentially the sober doctrine of that Hellenic philosopher who is generally known as the founder of logic and scientific method (*Met.*, Book λ, Ch. 7, and *Eth. Nik.*, Ch. 8).

MORRIS R. COHEN.

COLLEGE OF THE CITY OF NEW YORK.

Le Conflit de la Morale et de la Sociologie. Par SIMON DEPLOIGE. Louvain, Institut Supérieur de Philosophie, 1911.—pp. 424.

Durkheim and Lévy-Bruhl, representing the dominant French school of scientific sociology, declare that moral philosophy is a discredited study. "All that counts in ethics for genuine knowledge," says Lévy-Bruhl, "is investigation by sociological methods." This statement furnishes M. Deploige with the starting-point for a critical and historical review of scientific sociology, in particular that of Durkheim and his school. His object is to show, (1) that the conflict of ethics and sociology is an ancient conflict of ideas, long antedating the sociologists; (2) that the special criticism of ethics made by Durkheim and Lévy-Bruhl applies only to their predecessors in France, Rousseau and Cousin; (3) that the "scientific method" is by no means a modern discovery in ethics, since, not to speak of others, its essentials were long ago clearly outlined by Thomas Aquinas.